Physics

HP COMPUTER CURRICULUM

Mechanics

TEACHERS ADVISOR

HEWLETT PACKARD

Hewlett-Packard
Computer Curriculum Series

physics
TEACHER'S ADVISOR

mechanics

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This material is designed to be used with any Hewlett-Packard system with the BASIC programming language such as the 9830A, Educational BASIC, and the 2000 and 3000 series systems.

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INTRODUCTION

This Physics Problem-Solving Set consists of a Student Lab Book and the corresponding Teacher's Advisor. It was designed to help meet the need for computer-oriented problems in physics, providing students an opportunity to use a computer as a problem solving tool within a particular subject matter area.

The materials are designed for flexible use as desired by the individual instructor. The material and exercises in this unit are intended as an "enrichment" experience in the field of mechanics.

Nearly all the exercises involve calculus in a difference approximation. This enables a wide variety of problems to be undertaken which would be impossible using only algebra and trigonometry. However, the words calculus, derivative, and integration are never used. The student who uses this material will have a much better feeling for calculus when he studies it analytically. Of course the primary reason a student should use the material is to better understand the ideas in mechanics. The use of this material will not compete with your text. Instead, it can be used to supplement and enrich in any fashion you choose.

The degree of difficulty of the material is dependent upon the amount of assistance which you choose to provide. With no assistance, the better physics student should be challenged. However, given a good deal of assistance, any physics student should be able to work out the exercises with no great difficulty. The level of the material is determined by the assumption that students taking introductory physics will be quite capable as a group.

The Lab Books provide text material and programming exercises for the students, advanced problems, and a sample program. The Teacher's Advisor contains an example of a program to solve each exercise and a brief discussion of the important elements of exercise.

For best results, you should study all the solutions until you are certain you have a complete grasp of the general methods. This should be done before assigning the material to the class. Generally, the exercises are cumulative so that as techniques are developed they are used in subsequent exercises. Therefore, you will probably wish to proceed through the exercises in the order in which they are given.

The solutions should be treated as *typical* only. There are many ways to work a given problem. Students should be encouraged to develop their own solutions even though they may be quite different from those presented here. Also, you should not become obsessively concerned with computer efficiency. It is far more important that the student develop a logical and methodical approach to computer programming then to dwell on flashy techniques.

NOTES

RATES

Exercise 1 — Computing Velocity

This exercise utilizes the program from Figure 1 in the Student Lab Book. The only change required is a new DEF statement in line 150. The intent is to let the student see the limiting process as the average and instantaneous velocities approach each other. If the student tries a value of D too small, the accuracy begins to deteriorate due to internal round off error by the computer. If you choose, you might ask the student to investigate other values of time and plot the results.

```
100  REM PROGRAM FOR AVERAGE VELOCITY
110  PRINT "INPUT VALUE OF T DESIRED";
120  INPUT T
130  PRINT "INPUT VALUE OF D";
140  INPUT D
150  DEF FNA(T)=SIN(T)+T↑2
160  LET V=(FNA(T+D/2)-FNA(T-D/2))/D
170  PRINT "AVERAGE VELOCITY IS ";V
180  PRINT
190  GOTO 110
999  END
```

```
RUN

INPUT VALUE OF T DESIRED?1
INPUT VALUE OF D?.5
AVERAGE VELOCITY IS  2.53469

INPUT VALUE OF T DESIRED?1
INPUT VALUE OF D?.1
AVERAGE VELOCITY IS  2.54008

INPUT VALUE OF T DESIRED?1
INPUT VALUE OF D?.05
AVERAGE VELOCITY IS  2.54024

INPUT VALUE OF T DESIRED?1
INPUT VALUE OF D?.01
AVERAGE VELOCITY IS  2.5403

INPUT VALUE OF T DESIRED?2
INPUT VALUE OF D?.1
AVERAGE VELOCITY IS  3.58403

INPUT VALUE OF T DESIRED?2
INPUT VALUE OF D?.05
AVERAGE VELOCITY IS  3.58391

INPUT VALUE OF T DESIRED?2
INPUT VALUE OF D?.01
AVERAGE VELOCITY IS  3.584
```

Exercise 2 — Applications

This exercise is based upon Exercise 1, but a very important point is developed here. That is, with the computer we can handle cases that would be very difficult even if the student had completed a course in calculus. Assuming the student was acquainted with differentiation he would find great difficulty with (c). However it is handled in the same manner as (a) and (b).

```
150 DEF FNA(T)= 3*T↑3-4*T↑2+5
RUN


INPUT VALUE OF T DESIRED?1
INPUT VALUE OF D?1
AVERAGE VELOCITY IS  1.75

INPUT VALUE OF T DESIRED?1
INPUT VALUE OF D?.1
AVERAGE VELOCITY IS  1.00751

INPUT VALUE OF T DESIRED?1
INPUT VALUE OF D?.01
AVERAGE VELOCITY IS  1.00017

INPUT VALUE OF T DESIRED?2
INPUT VALUE OF D?.1
AVERAGE VELOCITY IS  20.0075

INPUT VALUE OF T DESIRED?2
INPUT VALUE OF D?.01
AVERAGE VELOCITY IS  20.0001


150 DEF FNA(T) = EXP(T)+T
RUN


INPUT VALUE OF T DESIRED?1
INPUT VALUE OF D?.1
AVERAGE VELOCITY IS  3.71942

INPUT VALUE OF T DESIRED?1
INPUT VALUE OF D?.01
AVERAGE VELOCITY IS  3.71833

INPUT VALUE OF T DESIRED?2
INPUT VALUE OF D?.1
AVERAGE VELOCITY IS  8.39216

INPUT VALUE OF T DESIRED?2
INPUT VALUE OF D?.01
AVERAGE VELOCITY IS  8.38928
```

```
150 DEF FNA(T) = COS(T↑3)-EXP(COS(T))
RUN

INPUT VALUE OF T DESIRED?1
INPUT VALUE OF D?1
AVERAGE VELOCITY IS -.633302

INPUT VALUE OF T DESIRED?1
INPUT VALUE OF D?.1
AVERAGE VELOCITY IS -1.08582

INPUT VALUE OF T DESIRED?1
INPUT VALUE OF D?.01
AVERAGE VELOCITY IS -1.08004

INPUT VALUE OF T DESIRED?2
INPUT VALUE OF D?.1
AVERAGE VELOCITY IS -10.5488

INPUT VALUE OF T DESIRED?2
INPUT VALUE OF D?.01
AVERAGE VELOCITY IS -11.2654

INPUT VALUE OF T DESIRED?2
INPUT VALUE OF D?.0075
AVERAGE VELOCITY IS -11.2685
```

Exercise 3 — Automatic Accuracy

This is an exercise in programming skill. The old value of the velocity is set equal to zero in line 140. The new value is computed in line 170. The test that is the heart of the process in line 180 gives the exit condition. Other than this, the program is straightforward.

```
100  REM AUTOMATIC VELOCITY PROGRAM
110  READ N
120  DEF FNA(T)=T↑2
130  FOR I=1 TO N
140  LET V0=0
150  LET D=1
160  READ T
170  LET V1=(FNA(T+D/2)-FNA(T-D/2))/D
180  IF ABS((V1-V0)/V1)<.00005 THEN 220
190  LET D=D/10
200  LET V0=V1
210  GOTO 170
220  PRINT "AT T = "T"   V =   "V1
230  NEXT I
800  DATA 4
801  DATA 1,2,3,4
999  END
```

```
RUN

AT T =  1       V =    2.
AT T =  2       V =    4.
AT T =  3       V =    6.00002
AT T =  4       V =    8.00001
```

Exercise 4 — Advanced — Plotting Results

Hopefully, the student will discover in this exercise that for a displacement that is a linear function of time, the velocity is a constant. You might assign the students other linear functions and let them discover the property of constant velocity.

```
120 DEF FNA(T) = T
RUN

AT T =  1        V =    .999999
AT T =  2        V =    1.
AT T =  3        V =    1.
AT T =  4        V =    1.

DONE
```

Exercise 5 — Rates and Trigonometric Functions

The student should discover that the desired function is the Cosine function. This is facilitated by plotting both the function for displacement and for velocity on the same graph.

```
100  REM AUTOMATIC VELOCITY PROGRAM
105  PRINT
106  PRINT " T","V1","SIN(T)","COS(T)"
107  PRINT
120  DEF FNA(T)=SIN(T)
130  FOR T=0 TO 1 STEP .1
140  LET V0=0
150  LET D=1
170  LET V1=(FNA(T+D/2)-FNA(T-D/2))/D
180  IF ABS(V1-V0)<.0001 THEN 220
190  LET D=D/10
200  LET V0=V1
210  GOTO 170
220  PRINT T,V1,SIN(T),COS(T)
230  NEXT T
999  END
```

RUN

T	V1	SIN(T)	COS(T)
0	1	0	1.
.1	.99498	9.98334E-02	.995004
.2	.980079	.198669	.980067
.3	.955343	.29552	.955336
.4	.92113	.389418	.921061
.5	.877619	.479426	.877582
.6	.825286	.564643	.825336
.7	.764847	.644218	.764842
.8	.696659	.717356	.696707
.9	.621557	.783327	.62161
1	.540257	.841471	.540302

DONE

FROM RATES TO DISPLACEMENTS

Exercise 6 — Displacement

This exercise introduces the student to the solution of differential equations. Of course, he won't know what a differential equation is; but due to the way the problem is phrased, the solution can be obtained without difficulty.

```
100  REM PROGRAM TO FIND DISPLACEMENT
110  REM GIVEN VELOCITY
120  PRINT "INPUT INITIAL VALUE OF X  ";
130  INPUT X0
140  PRINT "INPUT TIME INCREMENT  ";
150  INPUT D
160  DEF FNA(T)=COS(T)
170  FOR T=0 TO 1 STEP D
180  PRINT T,X0
190  LET X1=X0+FNA(T)*D
200  LET X0=X1
210  NEXT T
999  END
```

```
RUN


INPUT INITIAL VALUE OF X  ?0
INPUT TIME INCREMENT  ?.1
 0              0
 .1             .1
 .2             .1995
 .3             .297507
 .4             .393041
 .5             .485147
 .6             .572905
 .7             .655439
 .8             .731923
 .9             .801594
 1              .863755

DONE
```

Exercise 7 — Increasing the Accuracy

The notion of a function is illustrated very nicely in this exercise. With the definition of the cosine function in line 160, the average velocity can be clearly defined in line 190. The student should be aware of the fact that different values of displacement are obtained using the average velocity, and that these values are more accurate.

```
100   REM PROGRAM TO FIND DISPLACEMENT
110   REM GIVEN VELOCITY
120   PRINT "INPUT INITIAL VALUE OF X  ";
130   INPUT X0
140   PRINT "INPUT TIME INCREMENT  ";
150   INPUT D
160   DEF FNA(T)=COS(T)
170   FOR T=0 TO 1 STEP D
180   PRINT T,X0
190   LET X1=X0+((FNA(T+D)+FNA(T))/2)*D
200   LET X0=X1
210   NEXT T
999   END
```

```
RUN


INPUT INITIAL VALUE OF X  ?0
INPUT TIME INCREMENT  ?.1
 0              0
 .1             9.97502E-02
 .2             .198504
 .3             .295274
 .4             .389094
 .5             .479026
 .6             .564172
 .7             .643681
 .8             .716758
 .9             .782674
 1              .84077

DONE
```

Exercise 8 — Finding Displacement From Acceleration

This exercise needs to be related to the physics text to be most meaningful. The idea here is very powerful, and the students should work at it until the strategy is clear. The program is clear and needs no special explanation.

```
100  REM PROGRAM TO FIND DISPLACEMENT
110  REM GIVEN ACCELERATION
120  PRINT "INPUT INITIAL VALUE OF X  ";
130  INPUT XØ
140  PRINT "INPUT INITIAL VALUE OF V  ";
150  INPUT VØ
160  PRINT "INPUT TIME INCREMENT  ";
170  INPUT D
180  DEF FNA(T)=T
190  FOR T=Ø TO 1 STEP D
200  PRINT T,VØ,XØ
210  LET V1=VØ+FNA(T)*D
220  LET X1=XØ+VØ*D
230  LET VØ=V1
240  LET XØ=X1
250  NEXT T
999  END
```

```
RUN

INPUT INITIAL VALUE OF X  ?0
INPUT INITIAL VALUE OF V  ?0
INPUT TIME INCREMENT  ?.1
 0              0              0
 .1             0              0
 .2             .01            0
 .3             .03            .001
 .4             .06            .004
 .5             .1             .01
 .6             .15            .02
 .7             .21            .035
 .8             .28            .056
 .9             .36            .084
 1              .45            .12

DONE
```

Exercise 9 — Advanced — Average Values of Velocity and Acceleration

The only change in the program is to use average values of acceleration and velocity. This is done in lines 210 and 220. This is a good program to use in a heuristic sense. Supply the student with a wide variety of acceleration functions with initial conditions on velocity and position. Then the computer can be used to great advantage to investigate the characteristics of the motion. The student should be made aware that with the computer nearly any type of motion can be studied whereas only a few types of motion can be described if only algebra is used.

```
100   REM PROGRAM TO FIND DISPLACEMENT
110   REM GIVEN ACCELERATION
120   PRINT "INPUT INITIAL VALUE OF X  ";
130   INPUT X0
140   PRINT "INPUT INITIAL VALUE OF V  ";
150   INPUT V0
160   PRINT "INPUT TIME INCREMENT  ";
170   INPUT D
180   DEF FNA(T)=T
190   FOR T=0 TO 1 STEP D
200   PRINT T,V0,X0
210   LET V1=V0+((FNA(T+D)+FNA(T))/2)*D
220   LET X1=X0+((V1+V0)/2)*D
230   LET V0=V1
240   LET X0=X1
250   NEXT T
999   END


RUN


INPUT INITIAL VALUE OF X  ?0
INPUT INITIAL VALUE OF V  ?0
INPUT TIME INCREMENT  ?.1
 0              0              0
 .1             .005           .00025
 .2             .02            .0015
 .3             .045           .00475
 .4             .08            .011
 .5             .125           .02125
 .6             .18            .0365
 .7             .245           .05775
 .8             .32            .086
 .9             .405           .12225
 1              .5             .1675

DONE
```

Exercise 10 — Advanced — A Generalization

The only change required is in line 190 where the FOR statement is modified.

```
190 FOR T = 0 TO 4 STEP D
RUN

INPUT INITIAL VALUE OF X  ?0
INPUT INITIAL VALUE OF V  ?0
INPUT TIME INCREMENT  ?.1
 0              0              0
 .1             .005           .00025
 .2             .02            .0015
 .3             .045           .00475
 .4             .08            .011
 .5             .125           .02125
 .6             .18            .0365
 .7             .245           .05775
 .8             .32            .086
 .9             .405           .12225
 1              .5             .1675
 1.1            .605           .22275
 1.2            .72            .289
 1.3            .845           .36725
 1.4            .98            .4585
 1.5            1.125          .56375
 1.6            1.28           .684
 1.7            1.445          .82025
 1.8            1.62           .9735
 1.9            1.805          1.14475
 2.             2.             1.335
 2.1            2.205          1.54525
 2.2            2.42           1.7765
 2.3            2.645          2.02975
 2.4            2.88           2.306
 2.5            3.125          2.60625
 2.6            3.38           2.9315
 2.7            3.645          3.28275
 2.8            3.92           3.661
 2.9            4.205          4.06725
 3.             4.5            4.5025
```

3.1	4.805	4.96775
3.2	5.12	5.464
3.3	5.445	5.99225
3.4	5.78	6.5535
3.5	6.125	7.14875
3.6	6.48	7.779
3.7	6.845	8.44525
3.8	7.22	9.1485
3.9	7.605	9.88975
4.	8.	10.67

DONE

NEWTON'S SECOND LAW

Exercise 11 — Force and Motion

This exercise is designed to promote qualitative thinking about motion problems. It may help to have the student plot the results.

```
800 DATA 2,-4,2
RUN


T                 V                 X

 0                -4                 2
 1                -3.               -1.55
 2.               -2.               -4.1
 3.               -1.               -5.65
 4.               -2.83122E-06      -6.20001

DONE
```

```
800 DATA 2,-4,1
130 LET F = 8
RUN


T                 V                 X

 0                -4                 2
 1                 4.                1.6
 2.                12.               9.2
 3.                20.               24.8
 4.                28.               48.4

DONE
```

Exercise 12 — Converging on Accuracy

As the time increment in the problem decreases the student should begin to suspect that the velocity and position will be more accurate. You can run this program with some other values for force, mass, and initial conditions to give more experience.

```
801 DATA .1,10,5
RUN


T                    V                    X

 0                  -4                    2
 1                  -2.                  -1.1
 2.                 -1.84774E-06         -2.2
 3.                  2.                  -1.3
 4.                  4.                   1.59999

DONE
```

```
801 DATA .05,20,5
RUN


T                    V                    X

 0                  -4                    2
 1.                 -2.                  -1.05
 2.                 -2.83122E-06         -2.1
 3.                  2.                  -1.15001
 4.00001             3.99999              1.79999

DONE
```

```
801 DATA .01,100,5
RUN


T                  V                  X

 0                 -4                  2
 .999999           -2.                -1.01
 2.                -3.52412E-06       -2.02
 3.00002            1.99999           -1.03001
 4.00004            3.99999            1.95999

DONE
```

```
801 DATA .005,200,5
RUN


T                  V                  X

 0                 -4                  2
 .999999           -1.99995           -1.00498
 2.00002            4.37759E-05       -2.00993
 3.00004            2.00004           -1.01489
 4.00007            4.00009            1.98017

DONE
```

Exercise 13 — A Variable Force

A simple change in the program is required. Now the force is not a constant and must be computed each time step. This is done in line 185 in the program. It will be very easy for the student to respond to this simple change. You may wish to point out the vast difference in the analytical techniques produced by such a simple change. It is precisely at points such as this that the power of the computer becomes visible.

```
100  REM NEWTON'S SECOND LAW
110  READ X0,V0,M
120  READ D,N,L
140  PRINT
150  PRINT "T","V","X"
160  PRINT
170  LET C=N
180  FOR T=0 TO L STEP D
185  LET F=4*COS(T)
190  IF C<N THEN 220
200  PRINT T,V0,X0
210  LET C=0
220  LET V1=V0+F*D/M
230  LET X1=X0+V0*D
240  LET V0=V1
250  LET X0=X1
260  LET C=C+1
270  NEXT T
800  DATA 2,-4,1
801  DATA .1,10,5
999  END
```

```
RUN


T              V                X

 0             -4                2
 1             -.544982         -.305178
 2.            -8.26126E-02     -.322429
 3.            -3.03799         -1.52959
 4.            -6.69396         -6.31051

DONE
```

Exercise 14 — Advanced — A Challenge

This problem would be very difficult by analytical methods. However with the computer it merely requires several tests to locate the current value of displacement which in turn determines the appropriate value of force to use. The program follows without difficulty.

```
100  REM NEWTON'S SECOND LAW
110  READ X0,V0,M
120  READ D,N,L
140  PRINT
150  PRINT "T","V","X"
160  PRINT
170  LET C=N
180  FOR T=0 TO L STEP D
181  IF ABS(X0)<5 THEN 185
182  IF X0 >= 5 THEN 187
183  LET F=+4
184  GOTO 190
185  LET F=0
186  GOTO 190
187  LET F=-4
190  IF C<N THEN 220
200  PRINT T,V0,X0
210  LET C=0
220  LET V1=V0+F*D/M
230  LET X1=X0+V0*D
240  LET V0=V1
250  LET X0=X1
260  LET C=C+1
270  NEXT T
800  DATA 0,5,1
801  DATA .1,5,10
999  END
```

```
RUN


T              V              X

 0              5              0
 .5             5              2.5
 1              5              5
 1.5            3.             7.1
 2.             1.             8.2
 2.5           -.999999        8.3
 3.            -3.             7.4
 3.5           -5.             5.5
 4.            -5.8            2.72
 4.5           -5.8           -.179999
 5.            -5.8           -3.08
 5.5           -5.4           -5.98
 6.            -3.4           -8.28
 6.50001       -1.4           -9.58
 7.00001        .6            -9.88
 7.50001        2.6           -9.18
 8.00001        4.6           -7.48
 8.50001        6.6           -4.78
 9.00002        6.6           -1.48
 9.50002        6.6            1.82

DONE
```

Exercise 15 — Advanced — Barrier Penetration

The only difficult point in this exercise is to insure that each step produces a positive velocity as long as x is less than 5. A statement must be provided to test for this condition. If the test is failed the object is moving in the negative direction and will not penetrate. For your better students, this problem is a good one for which to develop an automatic program to replace the trial and error approach.

```
100  REM BARRIER SIMULATION
110  INPUT V0
115  LET A=V0
120  LET X0=0
130  LET V1=V0-5.00000E-02
140  LET X1=X0+1.00000E-02*V0
145  IF X1>X0 THEN 150
146  PRINT "VELOCITY TOO LOW"
147  GOTO 110
150  LET V0=V1
160  LET X0=X1
170  IF X0<5 THEN 130
180  PRINT
190  PRINT A,V0
195  PRINT
200  GOTO 110
999  END
```

```
RUN


?10

 10                7.05003

?5
VELOCITY TOO LOW
?7.5

 7.5               2.49998

?7
VELOCITY TOO LOW
?7.25

 7.25              1.64998

?7.125

 7.125             1.02498

?7.05

 7.05              .199983

?7.04
VELOCITY TOO LOW
?7.045
VELOCITY TOO LOW
?7.0475

 7.0475            .097482

?
DONE
```

HALF STEP METHOD

Exercise 16 — Half Step Computation

The half step is very easy to insert. This is done in line 175 in the program. Note that this does throw the velocity out of step. However the increase in accuracy for the position is dramatic. The values of position in the printout are exact with the exception of the last value. The exact value for t = 4 is 2.

```
100   REM NEWTON'S SECOND LAW
110   READ X0,V0,M
120   READ D,N,L
130   LET F=2
140   PRINT
150   PRINT "T","V","X"
160   PRINT
170   LET C=N
175   LET V0=V0+(F/M)*D/2
180   FOR T=0 TO L STEP D
190   IF C<N THEN 220
200   PRINT T,V0,X0
210   LET C=0
220   LET V1=V0+F*D/M
230   LET X1=X0+V0*D
240   LET V0=V1
250   LET X0=X1
260   LET C=C+1
270   NEXT T
800   DATA 2,-4,1
801   DATA .1,10,5
999   END
```

```
RUN


T               V               X

 0              -3.9             2
 1              -1.9            -1.
 2.              9.99982E-02    -2.
 3.              2.1            -1.
 4.              4.1             1.99999

DONE
```

Exercise 17 — Half Step Computation

This exercise uses the same program developed for Exercise 16 with the data used in Exercise 12.

```
801 DATA .1,10,5
RUN

T              V               X

 0             -3.9             2
 1             -1.9            -1.
 2.             9.99982E-02    -2.
 3.             2.1            -1.
 4.             4.1             1.99999

DONE
```

```
801 DATA .01,100,5
RUN

T              V               X

 0             -3.99            2
 .999999       -1.99           -1.
 2.             9.99671E-03    -2.
 3.00002        2.01           -1.00001
 4.00004        4.00999         1.99999

DONE
```

```
801 DATA .05,20,5
RUN

T              V               X

 0             -3.95            2
 1.            -1.95           -1.
 2.             4.99974E-02    -2.
 3.             2.05           -1.00001
 4.00001        4.05            1.99999

DONE
```

```
801 DATA .005,200,5
RUN

T              V               X

 0             -3.995           2
 .999999       -1.99495        -.999977
 2.00002        5.04391E-03    -1.99993
 3.00004        2.00504        -.999889
 4.00007        4.00509         2.00018

DONE
```

THE HARMONIC OSCILLATOR

Exercise 18 — Harmonic Oscillator

The program for this exercise poses no special difficulty. When the results are available, see if the students can see what kind of mathematical function would describe them. Generally, if the results are plotted the students will see the Sine and Cosine curves. This leads naturally to Exercise 19.

```
100  REM NEWTON'S SECOND LAW
110  READ X0,V0,M
120  READ N
130  LET P=3.14159
132  LET D=P/100
140  PRINT
150  PRINT "T","V","X"
160  PRINT
170  LET C=N
180  FOR T=0 TO 2*P STEP P/100
185  LET F=-X0
190  IF C<N THEN 220
200  PRINT T,V0,X0
210  LET C=0
220  LET V1=V0+F*D/M
230  LET X1=X0+V0*D
240  LET V0=V1
250  LET X0=X1
260  LET C=C+1
270  NEXT T
800  DATA 1,0,1
801  DATA 10
999  END
```

```
RUN


T                   V                   X

 0                   0                   1
 .314159            -.310446             .955791
 .628318            -.593443             .81716
 .942477            -.820892             .596802
 1.25664            -.969876             .315576
 1.5708             -1.02497             5.30496E-04
 1.88496            -.979821            -.317691
 2.19911            -.837879            -.607827
 2.51327            -.61214             -.841072
 2.82743            -.32397             -.993926
 3.14159            -1.08758E-03        -1.05056
 3.45575             .325103            -1.00445
 3.76991             .622559            -.859122
 4.08407             .861748            -.62787
 4.39823             1.01857            -.332587
 4.71239             1.07679            -1.67247E-03
 5.02655             1.02971             .332687
 5.34071             .880903             .637648
 5.65487             .644004             .882931
 5.96903             .341431             1.04383

DONE
```

Exercise 19 — Half Step Computation

The half step computation is in line 175.

```
100   REM NEWTON'S SECOND LAW
110   READ X0,V0,M
120   READ N
130   LET P=3.14159
132   LET D=P/100
140   PRINT
150   PRINT "T","V","X"
160   PRINT
170   LET C=N
175   LET V0=V0-(X0/M)*(D/2)
180   FOR T=0 TO 2*P STEP P/100
185   LET F=-X0
190   IF C<N THEN 220
200   PRINT T,V0,X0
210   LET C=0
220   LET V1=V0+F*D/M
230   LET X1=X0+V0*D
240   LET V0=V1
250   LET X0=X1
260   LET C=C+1
270   NEXT T
800   DATA 1,0,1
801   DATA 10
999   END
```

```
RUN


T                   V                   X

 0                 -.015708              1
 .314159           -.32546               .950915
 .628318           -.606279              .807838
 .942477           -.830266              .583908
 1.25664           -.974833              .300341
 1.5708            -1.02498             -1.55698E-02
 1.88496           -.97483              -.333082
 2.19911           -.828331             -.620989
 2.51327           -.598928             -.850687
 2.82743           -.308357             -.999014
 3.14159            1.54147E-02         -1.05058
 3.45575            .340881             -.999347
 3.76991            .636054             -.849342
 4.08407            .87161              -.614334
 4.39823            1.02379             -.316587
 4.71239            1.07682              1.52419E-02
 5.02655            1.02448              .348862
 5.34071            .870886              .651485
 5.65487            .630135              .893047
 5.96903            .325034              1.04919

DONE
```

Exercise 20 — Changing the Mass

This exercise should be correlated with the physics textbook for best results. Also, the student should plot the results to assist him to better observe the effects of the change in mass.

```
800 DATA 1,0,5
RUN


T                   V                   X

 0                  -3.14159E-03         1
 .314159            -6.57968E-02         .990141
 .628318            -.127278             .960732
 .942477            -.18637              .912315
 1.25664            -.241903             .845808
 1.5708             -.292774             .762486
 1.88496            -.33797              .66396
 2.19911            -.37659              .552142
 2.51327            -.407858             .429213
 2.82743            -.431143             .297575
 3.14159            -.44597              .159807
 3.45575            -.452029             1.86155E-02
 3.76991            -.449185            -.123223
 4.08407            -.437475            -.26291
 4.39823            -.417112            -.397688
 4.71239            -.388482            -.524888
 5.02655            -.352133            -.641987
 5.34071            -.308766            -.746653
 5.65487            -.259224            -.836799
 5.96903            -.20447             -.910618

DONE
```

```
800 DATA 1,0,2
RUN


T                    V                    X

 0                   -7.85398E-03          1
 .314159             -.16383               .975392
 .628318             -.312508              .902617
 .942477             -.446529              .785023
 1.25664             -.559217              .628179
 1.5708              -.644918              .439618
 1.88496             -.699276              .22847
 2.19911             -.719458              5.02351E-03
 2.51327             -.704303             -.219775
 2.82743             -.654381             -.43486
 3.14159             -.571979             -.629589
 3.45575             -.460997             -.79427
 3.76991             -.326758             -.920642
 4.08407             -.17576              -1.00229
 4.39823             -1.53575E-02         -1.03498
 4.71239              .146594             -1.01684
 5.02655              .302127             -.948531
 5.34071              .443546             -.833161
 5.65487              .563818             -.676183
 5.96903              .656912             -.485121

DONE
```

Exercise 21 — Advanced — The Pendulum

The goal in this exercise is to encourage the student to derive the equations below which describe the motion of a simple pendulum of length L.

$$\Delta\omega/\Delta t = -(g/L)\theta$$

$$\Delta\theta/\Delta t = \omega$$

Then, the solution can be obtained from the harmonic oscillator program.

NOTES

MORE COMPLICATED FORCES

Exercise 22 — Damped Harmonic Oscillator

The only change required is in the line specifying the force.

```
100  REM NEWTON'S SECOND LAW
110  READ X0,V0,M
120  READ N
130  LET P=3.14159
132  LET D=P/100
140  PRINT
150  PRINT "T","V","X"
160  PRINT
170  LET C=N
175  LET V0=V0-(X0/M)*(D/2)-(V0/M)*(D/2)
180  FOR T=0 TO 2*P STEP P/100
185  LET F=-X0-V0
190  IF C<N THEN 220
200  PRINT T,V0,X0
210  LET C=0
220  LET V1=V0+F*D/M
230  LET X1=X0+V0*D
240  LET V0=V1
250  LET X0=X1
260  LET C=C+1
270  NEXT T
800  DATA 1,0,1
801  DATA 10
999  END
```

```
RUN


T                    V                    X

 0                   -.015708              1
 .314159             -.280771              .95506
 .628318             -.451363              .840397
 .942477             -.53801               .684351
 1.25664             -.555616              .511266
 1.5708              -.521031              .340473
 1.88496             -.451085              .185967
 2.19911             -.361159              5.66313E-02
 2.51327             -.264255             -4.31658E-02
 2.82743             -.170514             -.112743
 3.14159             -8.71108E-02         -.154183
 3.45575             -1.84299E-02         -.171423
 3.76991              3.35695E-02         -.169421
 4.08407              6.88752E-02         -.153463
 4.39823              .088906             -.128625
 4.71239              9.60094E-02         -9.93906E-02
 5.02655              9.30146E-02         -6.94284E-02
 5.34071              .082862             -.041494
 5.65487              6.83225E-02         -1.74373E-02
 5.96903              5.18057E-02          1.71561E-03

DONE
```

Exercise 23 — An Experiment

This is very good combination laboratory and computer exercise.

Exercise 24 — Advanced — Driven, Damped Harmonic Oscillator

As in Exercise 22, all that is required is modification of the forces.

```
100  REM NEWTON'S SECOND LAW
110  READ X0,V0,M
120  READ N
130  LET P=3.14159
132  LET D=P/100
134  LET T=0
140  PRINT
150  PRINT "T","V","X"
160  PRINT
170  LET C=N
175  LET V0=V0-(X0/M)*(D/2)-(V0/M)*(D/2)+(2*COS(T)/M)*(D/2)
180  FOR T=0 TO 2*P STEP P/100
185  LET F=-X0-V0+2*COS(T)
190  IF C<N THEN 220
200  PRINT T,V0,X0
210  LET C=0
220  LET V1=V0+F*D/M
230  LET X1=X0+V0*D
240  LET V0=V1
250  LET X0=X1
260  LET C=C+1
270  NEXT T
800  DATA 1,0,1
801  DATA 10
999  END
```

```
RUN


T                   V                   X

 0                   .015708             1
 .314159             .272497             1.04444
 .628318             .386542             1.15026
 .942477             .338567             1.26912
 1.25664             .13793              1.35074
 1.5708             -.182084             1.35134
 1.88496            -.571079             1.24018
 2.19911            -.969553             1.00389
 2.51327            -1.31718             .64809
 2.82743            -1.56052             .196585
 3.14159            -1.65937            -.311768
 3.45575            -1.59135            -.827912
 3.76991            -1.35415            -1.29856
 4.08407            -.965564            -1.67259
 4.39823            -.461242            -1.90704
 4.71239             .109376            -1.97227
 5.02655             .688574            -1.85547
 5.34071             1.21664            -1.56242
 5.65487             1.63829            -1.11719
 5.96903             1.9085             -.559911

DONE
```

ORBITAL MOTION

Exercise 25 — Orbital Motion

This problem should cause no particular difficulty. The symbol assignment must be given particular attention however. The solution given contains a half step computation in the velocity.

```
100  REM ORBITAL PROBLEM
110  READ X1,X2,V1,V2
111  READ D,N,L
120  LET R1=(X1↑2+X2↑2)↑1.5
121  LET V1=V1-(X1*D)/(R1*2)
122  LET V2=V2-(X2*D)/(R1*2)
125  LET C=N
130  FOR T=0 TO L STEP D
135  IF C<N THEN 151
140  PRINT T,X1,X2,R1
141  LET C=0
151  LET X1=X1+V1*D
152  LET X2=X2+V2*D
153  LET R1=(X1↑2+X2↑2)↑1.5
154  LET V1=V1-X1*D/R1
155  LET V2=V2-X2*D/R1
156  LET C=C+1
157  NEXT T
800  DATA 1,0,0,1
801  DATA 1.00000E-02,10,1
999  END
```

```
RUN

 0            1            0              1.
 .1           .995004      9.98351E-02    1.
 .2           .980066      .198673        1.
 .3           .955336      .295525        1.
 .4           .921061      .389425        1.00001
 .5           .877582      .479433        1.00001
 .6           .825335      .564652        1.00001
 .7           .764841      .644228        1.00002
 .799999      .696706      .717368        1.00002
 .899999      .621609      .78334         1.00003
 .999999      .540301      .841485        1.00003

DONE
```

Exercise 26 — Experimentation

This is strictly a *discovery* exercise. Students should be encouraged to try a wide range of initial positions and velocities and plot the resultant orbits. See if they can predict the orbit that will result from given initial conditions.

Exercise 27 — A Challenge

The initial conditions place the object at rest under the influence of a central force. As indicated in the printout, the object begins to acquire velocity towards the center of force as would be expected. The unexpected part is how can the object arrive at a point further out on the other side than its initial distance? The problem is caused by the large changes in velocity that take place as the object nears the center of force. These large changes invalidate the whole solution. Students must constantly be on the alert for this type of error.

```
800 DATA 1,0,0,0
801 DATA .01,10,1.5
RUN

 0                1                0          1.
 .1               .994992          0          .985051
 .2               .979865          0          .940804
 .3               .954303          0          .869078
 .4               .917733          0          .772946
 .5               .869252          0          .656805
 .6               .807476          0          .526488
 .7               .730258          0          .389429
 .799999          .634073          0          .254929
 .899999          .512477          0          .134593
 .999999          .350752          0          4.31519E-02
 1.1              8.03883E-02      0          5.19491E-04
 1.2             -2.13429          0          9.72217
 1.3             -4.52498          0          92.6509
 1.4             -6.91509          0          330.669
 1.5             -9.30498          0          805.65

DONE
```

Exercise 28 — Advanced — A Double Force Center

The solution is obtained by adding an additional force to that considered previously. One of the fixed masses is located at (W,O) with the other at (-W,O). This program is fun to play with. Students can discover very interesting orbits and effects.

```
100  REM DOUBLE FORCE CENTER
110  READ X1,X2,V1,V2
115  READ D,N,L
116  READ W
120  LET R1=((X1-W)↑2+X2↑2)↑1.5
130  LET R2=((X1+W)↑2+X2↑2)↑1.5
140  LET V1=V1-(X1-W)*D/(R1*2)-(X1+W)*D/(R2*2)
150  LET V2=V2-(X2*D)/(R1*2)-(X2*D)/(R2*2)
160  LET C=N
170  FOR T=0 TO L STEP D
180  IF C<N THEN 210
190  PRINT T,X1,X2
200  LET C=0
210  LET X1=X1+V1*D
220  LET X2=X2+V2*D
230  LET R1=((X1-W)↑2+X2↑2)↑1.5
240  LET R2=((X1+W)↑2+X2↑2)↑1.5
250  LET V1=V1-(X1-W)*D/R1-(X1+W)*D/R2
260  LET V2=V2-(X2*D)/R1-(X2*D)/R2
270  LET C=C+1
280  NEXT T
800  DATA 1.5,0,0,1.5
801  DATA 1.00000E-02,10,2
802  DATA 1
999  END
```

RUN

0	1.5	0
.1	1.47937	.148018
.2	1.41946	.28435
.3	1.32587	.398869
.4	1.20676	.48417
.5	1.07155	.536144
.6	.929619	.553918
.7	.789188	.539354
.799999	.656693	.496284
.899999	.536439	.429703
.999999	.430599	.345035
1.1	.339437	.247546
1.2	.261709	.141963
1.3	.195149	3.22819E-02
1.4	.136995	-7.82628E-02
1.5	8.44511E-02	-.187123
1.6	3.50322E-02	-.292324
1.7	-1.32462E-02	-.392354
1.8	-6.18257E-02	-.486073
1.9	-.111659	-.572639
2.	-.163308	-.65144

DONE